

SOME NEW PARANORMED SEQUENCE SPACES AND α - CORE

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ABSTRACT. In this study, we define new paranormed sequence spaces by combining a double sequential band matrix and a diagonal matrix. Furthermore, we compute the α -, β - and γ - duals and obtain bases for these sequence spaces. Besides this, we characterize the matrix transformations from the new paranormed sequence spaces to the spaces $c_0(q)$, $c(q)$, $\ell(q)$ and $\ell_\infty(q)$. Finally, α -core of a complex-valued sequence has been introduced, and we prove some inclusion theorems related to this new type of core.

1. INTRODUCTION

By ω , we shall denote the space of all real valued sequences. Any vector subspace of ω is called as a *sequence space*. We shall write ℓ_∞, c and c_0 for the spaces of all bounded, convergent and null sequences, respectively. Also by bs, cs, ℓ_1 and ℓ_p ; we denote the spaces of all bounded, convergent, absolutely and p - absolutely convergent series, respectively; $1 < p < \infty$.

A linear topological space X over the real field \mathbb{R} is said to be a paranormed space if there is a subadditive function $g : X \rightarrow \mathbb{R}$ such that $g(\theta) = 0, g(x) = g(-x)$ and scalar multiplication is continuous, i.e., $|\alpha_n - \alpha| \rightarrow 0$ and $g(x_n - x) \rightarrow 0$ imply $g(\alpha_n x_n - \alpha x) \rightarrow 0$ for all α 's in \mathbb{R} and all x 's in X , where θ is the zero vector in the linear space X .

Assume here and after that (p_k) be a bounded sequences of strictly positive real numbers with $\sup p_k = H$ and $M = \max\{1, H\}$. Then, the linear spaces $c(p), c_0(p), \ell_\infty(p)$ and $\ell(p)$ were defined by Maddox [9, 10] (see also Simons [12] and Nakano [11]) as follows:

$$\begin{aligned} c(p) &= \left\{ x = (x_k) \in \omega : \lim_{k \rightarrow \infty} |x_k - l|^{p_k} = 0 \text{ for some } l \in \mathbb{C} \right\}, \\ c_0(p) &= \left\{ x = (x_k) \in \omega : \lim_{k \rightarrow \infty} |x_k|^{p_k} = 0 \right\}, \\ \ell_\infty(p) &= \left\{ x = (x_k) \in \omega : \sup_{k \in \mathbb{N}} |x_k|^{p_k} < \infty \right\} \end{aligned}$$

and

$$\ell(p) = \left\{ x = (x_k) \in \omega : \sum_k |x_k|^{p_k} < \infty \right\},$$

which are the complete spaces paranormed by

$$h_1(x) = \sup_{k \in \mathbb{N}} |x_k|^{p_k/M} \text{ iff } \inf_{p_k} > 0 \quad \text{and} \quad h_2(x) = \left(\sum_k |x_k|^{p_k} \right)^{1/M},$$

respectively. We shall assume throughout that $p_k^{-1} + (p'_k)^{-1} = 1$ provided $1 < \inf p_k < H < \infty$. For simplicity in notation, here and in what follows, the summation without limits runs from 0 to ∞ . By \mathcal{F} and \mathbb{N}_k , we shall denote the collection of all finite subsets of \mathbb{N} and the set of all $n \in \mathbb{N}$ such that $n \geq k$, respectively.

For the sequence spaces X and Y , define the set $S(X, Y)$ by

$$(1.1) \quad S(X, Y) = \{z = (z_k) : xz = (x_k z_k) \in Y \text{ for all } x \in X\}.$$

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With the notation of (1.1), the α -, β - and γ - duals of a sequence space X , which are respectively denoted by X^α , X^β and X^γ , are defined by

$$X^\alpha = S(X, \ell_1), \quad X^\beta = S(X, cs) \text{ and } X^\gamma = S(X, bs).$$

Let (X, h) be a paranormed space. A sequence (b_k) of the elements of X is called a basis for X if and only if, for each $x \in X$, there exists a unique sequence (α_k) of scalars such that

$$h \left(x - \sum_{k=0}^n \alpha_k b_k \right) \rightarrow 0 \text{ as } n \rightarrow \infty.$$

The series $\sum \alpha_k b_k$ which has the sum x is then called the expansion of x with respect to (b_n) and written as $x = \sum \alpha_k b_k$.

Let X, Y be any two sequence spaces and $A = (a_{nk})$ be an infinite matrix of real numbers a_{nk} , where $n, k \in \mathbb{N}$. Then, we say that A defines a matrix mapping from X into Y , and we denote it by writing $A : X \rightarrow Y$, if for every sequence $x = (x_k) \in X$ the sequence $Ax = ((Ax)_n)$, the A -transform of x , is in Y , where

$$(1.2) \quad (Ax)_n = \sum_k a_{nk} x_k, \quad (n \in \mathbb{N}).$$

By $(X : Y)$, we denote the class of all matrices A such that $A : X \rightarrow Y$. Thus, $A \in (X : Y)$ if and only if the series on the right-hand side of (1.2) converges for each $n \in \mathbb{N}$ and every $x \in X$, and we have $Ax = \{(Ax)_n\}_{n \in \mathbb{N}} \in Y$ for all $x \in X$. A sequence x is said to be A -summable to α if Ax converges to α which is called as the A -limit of x . If X and Y are equipped with the limits X -lim and Y -lim, respectively, $A \in (X : Y)$ and Y -lim $_n A_n(x) = X$ -lim $_k x_k$ for all $x \in X$, then we say that A regularly maps X into Y and write $A \in (X : Y)_{reg}$.

Let $x = (x_k)$ be a sequence in \mathbb{C} , the set of all complex numbers, and R_k be the least convex closed region of complex plane containing $x_k, x_{k+1}, x_{k+2}, \dots$. The Knopp Core (or \mathcal{K} -core) of x is defined by the intersection of all R_k ($k=1, 2, \dots$), (see [14], pp.137). In [15], it is shown that

$$\mathcal{K} - core(x) = \bigcap_{z \in \mathbb{C}} B_x(z)$$

for any bounded sequence x , where $B_x(z) = \{w \in \mathbb{C} : |w - z| \leq \limsup_k |x_k - z|\}$.

Let E be a subset of \mathbb{N} . The natural density δ of E is defined by

$$\delta(E) = \lim_n \frac{1}{n} |\{k \leq n : k \in E\}|$$

where $|\{k \leq n : k \in E\}|$ denotes the number of elements of E not exceeding n . A sequence $x = (x_k)$ is said to be statistically convergent to a number l , if $\delta(\{k : |x_k - l| \geq \varepsilon\}) = 0$ for every ε . In this case we write $st - \lim x = l$, [17]. By st we denote the space of all statistically convergent sequences.

In [18], the notion of the statistical core (or st -core) of a complex valued sequence has been introduced by Fridy and Orhan and it is shown for a statistically bounded sequence x that

$$st - core(x) = \bigcap_{z \in \mathbb{C}} C_x(z),$$

where $C_x(z) = \{w \in \mathbb{C} : |w - z| \leq st - \limsup_k |x_k - z|\}$. The core theorems have been studied by many authors. For instance see [20, 21, 22, 23, 24] and the others.

We write $\mathcal{U} = \{u \in \omega : u_k \neq 0 \text{ for all } k\}$ and $\mathcal{U}^+ = \{u \in \omega : u_k > 0 \text{ for all } k\}$; if $u \in \mathcal{U}$ then we write $1/u = (1/u_k)$ where $k \in \mathbb{N}$. By e and $e^{(n)}$ ($n = 0, 1, 2, \dots$), we denote the sequences such that $e_k = 1$ for $k = 0, 1, \dots$, and $e_n^{(n)} = 1$ and $e_k^{(n)} = 0$ for $k \neq n$.

An infinite matrix $T = (t_{nk})$ is said to be a triangle if $t_{nk} = 0$ ($k > n$) and $t_{nn} \neq 0$ for all n . Let us give the definition of some triangle limitation matrices which are needed in the text. Let $t = (t_k)$ be a sequence of positive reals and write

$$Q_n = \sum_{k=0}^n t_k, \quad (n \in \mathbb{N}).$$

Then the Cesàro mean of order one, Riesz mean with respect to the sequence $t = (t_k)$ and A_r- mean with $0 < r < 1$ are respectively defined by the matrices $C = (c_{nk})$, $R^t = (r_{nk}^t)$ and $B(r, s) = \{b_{nk}(r, s)\}$; where

$$c_{nk} = \begin{cases} \frac{1}{n+1}, & (0 \leq k \leq n), \\ 0, & (k > n), \end{cases} \quad r_{nk}^t = \begin{cases} \frac{t_k}{Q_n}, & (0 \leq k \leq n), \\ 0, & (k > n), \end{cases}$$

and

$$b_{nk}(r, s) = \begin{cases} r, & (k = n), \\ s, & (k = n - 1), \\ 0, & \text{otherwise} \end{cases}$$

for all $k, n \in \mathbb{N}$ and $r, s \in \mathbb{R} \setminus \{0\}$. Additionally, define the summation $S = (s_{nk})$ and the difference matrix $\Delta^{(1)} = (\delta_{nk})$ and the double sequential band matrix $\tilde{B} = B(\tilde{r}, \tilde{s}) = \{b_{nk}(\tilde{r}, \tilde{s})\}$ by

$$s_{nk} = \begin{cases} 1, & (0 \leq k \leq n), \\ 0, & (k > n), \end{cases} \quad \text{and} \quad \delta_{nk} = \begin{cases} (-1)^{n-k}, & (n - 1 \leq k \leq n), \\ 0, & (0 \leq k < n - 1 \text{ or } k > n), \end{cases}$$

and

$$b_{nk}(\tilde{r}, \tilde{s}) = \begin{cases} r_k, & (k = n), \\ s_k, & (k = n - 1), \\ 0, & \text{otherwise} \end{cases}$$

for all $k, n \in \mathbb{N}$.

Defining the diagonal matrix $D = (d_{nn})$ by $d_{nn} = 1/\alpha_n$ for $n = 0, 1, \dots$ and $\alpha = (\alpha_n) \in \mathcal{U}^+$ and putting $\tilde{T} = DB(\tilde{r}, \tilde{s})$.

The main purpose of this study is to introduce the paranormed sequence spaces $s_\alpha^0(\tilde{B}, p)$, $s_\alpha^{(c)}(\tilde{B}, p)$, $s_\alpha^{(\infty)}(\tilde{B}, p)$ and $\ell_\alpha(\tilde{B}, p)$ which are the set of all sequences whose \tilde{T} -transforms are in the spaces $c_0(p)$, $c(p)$, $\ell_\infty(p)$ and $\ell(p)$, respectively; where \tilde{T} denotes the matrix $\tilde{T} = DB(\tilde{r}, \tilde{s}) = \{t_{nk}(\tilde{r}, \tilde{s}, \alpha)\}$ defined by

$$t_{nk}(\tilde{r}, \tilde{s}, \alpha) = \begin{cases} r_n/\alpha_n, & (k = n), \\ s_{n-1}/\alpha_n, & (k = n - 1), \\ 0 & \text{otherwise.} \end{cases}$$

Also, we have investigated some topological structures, which have completeness, the $\alpha-$, $\beta-$ and $\gamma-$ duals, and the bases of these sequence spaces. Besides this, we characterize some matrix mappings on these spaces. Finally, we have defined $\alpha-$ core of a sequence and characterized some class of matrices for which $\alpha - \text{core}(Ax) \subseteq \mathcal{K} - \text{core}(x)$ and $\alpha - \text{core}(Ax) \subseteq st_A - \text{core}(x)$ for all $x \in \ell_\infty$.

2. THE PARANORMED SEQUENCE SPACES $\lambda(\tilde{B}, p)$ FOR $\lambda \in \{s_\alpha^0, s_\alpha^{(c)}, s_\alpha^{(\infty)}, \ell_\alpha\}$

In this section, we define the new sequence spaces $\lambda(\tilde{B}, p)$ for $\lambda \in \{s_\alpha^0, s_\alpha^{(c)}, s_\alpha^{(\infty)}, \ell_\alpha\}$ derived by using the double sequential band matrix and the diagonal matrix, and prove that these sequence spaces are the complete paranormed linear metric spaces and compute their $\alpha-$, $\beta-$ and $\gamma-$ duals. Moreover, we give the basis for the spaces $\lambda(\tilde{B}, p)$ for $\lambda \in \{s_\alpha^0, s_\alpha^{(c)}, \ell_\alpha\}$.

For a sequence space X , the matrix domain X_A of an infinite matrix A is defined by

$$(2.1) \quad X_A = \{x = (x_k) \in \omega : Ax \in X\}.$$

In [5], Choudhary and Mishra have defined the sequence space $\overline{\ell(p)}$ which consists of all sequences such that S -transforms are in $\ell(p)$, where $S = (s_{nk})$ is defined by

$$s_{nk} = \begin{cases} 1, & (0 \leq k \leq n), \\ 0, & (k > n). \end{cases}$$

Başar and Altay [3] have recently examined the space $bs(p)$ which is formerly defined by Başar in [4] as the set of all series whose sequences of partial sums are in $\ell_\infty(p)$. More recently, Altay and Başar have studied the sequence spaces $r^t(p)$, $r_\infty^t(p)$ in [1] and $r_c^t(p)$, $r_0^t(p)$ in [2] which are derived by the Riesz means from the sequence spaces $\ell(p)$, $\ell_\infty(p)$, $c(p)$ and $c_0(p)$ of Maddox, respectively. With the notation of (2.1), the spaces $\overline{\ell(p)}$, $bs(p)$, $r^t(p)$, $r_\infty^t(p)$, $r_c^t(p)$ and $r_0^t(p)$ may be redefined by

$$\overline{\ell(p)} = [\ell(p)]_S, \quad bs(p) = [\ell_\infty(p)]_S, \quad r^t(p) = [\ell(p)]_{R^t},$$

$$r_\infty^t(p) = [\ell_\infty(p)]_{R^t}, \quad r_c^t(p) = [c(p)]_{R^t}, \quad r_0^t(p) = [c_0(p)]_{R^t}.$$

It is well known that paranormed spaces have more general properties than normed spaces. In the literature, the approach of constructing a new sequence space on the paranormed space by means of the matrix domain of a particular limitation method has recently been employed by several authors, e.g., Yeşilkayagil and Başar [25], Nergiz and Başar [26, 27], Karakaya and Polat [28], Özger and Başar [29].

The domain of the matrix $B(r, s)$ in the classical spaces ℓ_∞, c_0 and c has recently been studied by Kirişçi and Başar in [36]. The characterizations of compact matrix operators between some of those spaces were given by Djolović in [37]. Recently difference sequence spaces have extensively been studied, for instance in [30, 31, 32, 33, 34, 35].

The sequence spaces $s_\alpha^0, s_\alpha^{(c)}, s_\alpha^{(\infty)}$ and ℓ_α , where $\alpha = (\alpha_n) \in \mathcal{U}^+$ were introduced by de Malafosse and Rakočević in [38, 39] as follows:

$$\begin{aligned} s_\alpha^0 &= \left\{ x = (x_k) \in \omega : \lim_{k \rightarrow \infty} \frac{x_k}{\alpha_k} = 0 \right\} \\ s_\alpha^{(c)} &= \left\{ x = (x_k) \in \omega : \exists l \in \mathbb{C} \ni \lim_{k \rightarrow \infty} \left(\frac{x_k}{\alpha_k} - l \right) = 0 \right\} \\ s_\alpha^{(\infty)} &= \left\{ x = (x_k) \in \omega : \sup_{k \in \mathbb{N}} \left| \frac{x_k}{\alpha_k} \right| < \infty \right\} \\ \ell_\alpha &= \left\{ x = (x_k) \in \omega : \sum_k \left| \frac{x_k}{\alpha_k} \right|^p < \infty \right\} \end{aligned}$$

These sequence spaces have extensively been examined by B de Malafosse in [40, 41, 42, 43].

Following Choudhary and Mishra [5], Başar and Altay [3], Altay and Başar [1, 2], we define the sequence spaces $\lambda(\tilde{B}, p)$ for $\lambda \in \{s_\alpha^0, s_\alpha^{(c)}, s_\alpha^{(\infty)}, \ell_\alpha\}$ by

$$\lambda(\tilde{B}, p) = \left\{ x = (x_k) \in \omega : \left(\frac{r_k x_k + s_{k-1} x_{k-1}}{\alpha_k} \right) \in \mu(p) \right\}.$$

for all $k \in \mathbb{N}$ and $\mu \in \{c_0, c, \ell_\infty, \ell\}$. With the notation (2.1), we may redefine the spaces $s_\alpha^0(\tilde{B}, p), s_\alpha^{(c)}(\tilde{B}, p), s_\alpha^{(\infty)}(\tilde{B}, p)$ and $\ell_\alpha(\tilde{B}, p)$ as follows:

$$\begin{aligned} s_\alpha^0(\tilde{B}, p) &= \{c_0(p)\}_{\tilde{T}}, \quad s_\alpha^{(c)}(\tilde{B}, p) = \{c(p)\}_{\tilde{T}}, \\ s_\alpha^{(\infty)}(\tilde{B}, p) &= \{\ell_\infty(p)\}_{\tilde{T}}, \quad \ell_\alpha(\tilde{B}, p) = \{\ell(p)\}_{\tilde{T}}. \end{aligned}$$

In the case $p = e$, the sequence space $\lambda(\tilde{B}, p)$ is reduced to $\lambda(\tilde{B})$ which is introduced by E. Malkowsky et al. [44] for $\lambda \in \{s_\alpha^0, s_\alpha^{(c)}, s_\alpha^{(\infty)}\}$. On the other hand, it is clear that Δ can be obtained as a special case of $B(\tilde{r}, \tilde{s})$ for $\tilde{r} = e$ and $\tilde{s} = -e$ and it is also trivial that $B(\tilde{r}, \tilde{s})$ reduces to $B(r, s)$ in the special case $\tilde{r} = re$ and $\tilde{s} = se$. So, the results related to the domain of the matrix $B(\tilde{r}, \tilde{s})$ are more general and more comprehensive than the corresponding ones of the domains of the matrices Δ and $B(r, s)$.

Define the sequence $y = (y_n)$, which will be frequently used as the $\tilde{T} = DB(\tilde{r}, \tilde{s})$ -transform of a sequence $x = (x_n)$, i.e.

$$(2.2) \quad y_n = \tilde{T}_n x = \frac{r_n x_n + s_{n-1} x_{n-1}}{\alpha_n}; \quad (n \in \mathbb{N}).$$

Since the proof may also be obtained in the similar way as for the other spaces, to avoid the repetition of the similar statements, we give the proof only for one of those spaces. Now, we may begin with the following theorem which is essential in the study.

Theorem 2.1. (i) The sequence spaces $\lambda(\tilde{B}, p)$ for $\lambda \in \{s_\alpha^0, s_\alpha^{(c)}, s_\alpha^{(\infty)}\}$ are the complete linear metric spaces paranormed by g , defined by

$$g(x) = \sup_{k \in \mathbb{N}} \left| \frac{r_k x_k + s_{k-1} x_{k-1}}{\alpha_k} \right|^{p_k/M}.$$

g is a paranorm for the spaces $s_\alpha^{(c)}(\tilde{B}, p)$ and $s_\alpha^{(\infty)}(\tilde{B}, p)$ only in the trivial case $\inf p_k > 0$.
(ii) $\ell_\alpha(\tilde{B}, p)$ is a complete linear metric space paranormed by

$$g^*(x) = \left(\sum_k \left| \frac{r_k x_k + s_{k-1} x_{k-1}}{\alpha_k} \right|^{p_k} \right)^{1/M}.$$

Proof. We prove the theorem for the space $s_\alpha^{(0)}(\tilde{B}, p)$. The linearity of $s_\alpha^{(0)}(\tilde{B}, p)$ with respect to the coordinatewise addition and scalar multiplication follows from the following inequalities which are satisfied for $x, z \in s_\alpha^{(0)}(\tilde{B}, p)$ (see [8, p.30]):

$$(2.3) \quad \sup_{k \in \mathbb{N}} \left| \frac{r_k(x_k + z_k) + s_{k-1}(x_{k-1} + z_{k-1})}{\alpha_k} \right|^{p_k/M} \leq \sup_{k \in \mathbb{N}} \left| \frac{r_k x_k + s_{k-1} x_{k-1}}{\alpha_k} \right|^{p_k/M} + \sup_{k \in \mathbb{N}} \left| \frac{r_k z_k + s_{k-1} z_{k-1}}{\alpha_k} \right|^{p_k/M}$$

and for any $\beta \in \mathbb{R}$ (see [10]),

$$(2.4) \quad |\beta|^{p_k} \leq \max\{1, |\beta|^M\}.$$

It is clear that $g(\theta) = 0$ and $g(x) = g(-x)$ for all $x \in s_\alpha^{(0)}(\tilde{B}, p)$. Again the inequalities (2.3) and (2.4) yield the subadditivity of g and

$$g(\beta x) \leq \max\{1, |\beta|\} g(x).$$

Let $\{x^n\}$ be any sequence of the points $x^n \in s_\alpha^{(0)}(\tilde{B}, p)$ such that $g(x^n - x) \rightarrow 0$ and (β_n) also be any sequence of scalars such that $\beta_n \rightarrow \beta$. Then, since the inequality

$$g(x^n) \leq g(x) + g(x^n - x)$$

holds by the subadditivity of g , $\{g(x^n)\}$ is bounded and we thus have

$$\begin{aligned} g(\beta_n x^n - \beta x) &= \sup_{k \in \mathbb{N}} \left| \frac{r_k(\beta_n x_k^n - \beta x_k) - s_{k-1}(\beta_n x_{k-1}^n - \beta x_{k-1})}{\alpha_k} \right|^{p_k/M} \\ &\leq |\beta_n - \beta| g(x^n) + |\beta| g(x^n - x), \end{aligned}$$

which tends to zero as $n \rightarrow \infty$. That is to say that the scalar multiplication is continuous. Hence, g is a paranorm on the space $s_\alpha^{(0)}(\tilde{B}, p)$.

It remains to prove the completeness of the space $s_\alpha^{(0)}(\tilde{B}, p)$. Let $\{x^i\}$ be any Cauchy sequence in the space $s_\alpha^{(0)}(\tilde{B}, p)$, where $x^i = \{x_0^{(i)}, x_1^{(i)}, x_2^{(i)}, \dots\}$. Then, for a given $\varepsilon > 0$ there exists a positive integer $n_0(\varepsilon)$ such that

$$g(x^i - x^j) < \frac{\varepsilon}{2}$$

for all $i, j \geq n_0(\varepsilon)$. We obtain by using definition of g for each fixed $k \in \mathbb{N}$ that

$$(2.5) \quad \begin{aligned} |\{\tilde{T}x^i\}_k - \{\tilde{T}x^j\}_k|^{p_k/M} &\leq \sup_{k \in \mathbb{N}} |\{\tilde{T}x^i\}_k - \{\tilde{T}x^j\}_k|^{p_k/M} \\ &< \frac{\varepsilon}{2} \end{aligned}$$

for every $i, j \geq n_0(\varepsilon)$, which leads us to the fact that $\{(\tilde{T}x^0)_k, (\tilde{T}x^1)_k, \dots\}$ is a Cauchy sequence of real numbers for every fixed $k \in \mathbb{N}$. Since \mathbb{R} is complete, it converges, say

$$\{\tilde{T}x^i\}_k \rightarrow \{\tilde{T}x\}_k$$

as $i \rightarrow \infty$. Using these infinitely many limits $(\tilde{T}x)_0, (\tilde{T}x)_1, \dots$, we define the sequence $\{(\tilde{T}x)_0, (\tilde{T}x)_1, \dots\}$. We have from (2.5) with $j \rightarrow \infty$ that

$$(2.6) \quad |\{\tilde{T}x^i\}_k - \{\tilde{T}x\}_k|^{p_k/M} \leq \frac{\varepsilon}{2} \quad (i \geq n_0(\varepsilon))$$

for every fixed $k \in \mathbb{N}$. Since $x^i = \{x_k^{(i)}\} \in s_\alpha^{(0)}(\tilde{B}, p)$,

$$|\{\tilde{T}x^i\}_k|^{p_k/M} < \frac{\varepsilon}{2}$$

for all $k \in \mathbb{N}$. Therefore, we obtain (2.6) that

$$\begin{aligned} |\{\tilde{T}x\}_k|^{p_k/M} &\leq |\{\tilde{T}x\}_k - \{\tilde{T}x^i\}_k|^{p_k/M} + |\{\tilde{T}x^i\}_k|^{p_k/M} \\ &< \varepsilon \quad (i \geq n_0(\varepsilon)). \end{aligned}$$

This shows that the sequence $\{\tilde{T}x\}$ belongs to the space $c_0(p)$. Since $\{x^i\}$ was an arbitrary Cauchy sequence, the space $s_\alpha^{(0)}(\tilde{B}, p)$ is complete and this concludes the proof. \square

Theorem 2.2. *The sequence spaces $s_\alpha^{(\infty)}(\tilde{B}, p)$, $s_\alpha^{(c)}(\tilde{B}, p)$, $s_\alpha^0(\tilde{B}, p)$ and $\ell_\alpha(\tilde{B}, p)$ are linearly isomorphic to the spaces $\ell_\infty(p)$, $c(p)$, $c_0(p)$ and $\ell(p)$, respectively, where $0 < p_k \leq H < \infty$.*

Proof. We establish this for the space $s_\alpha^{(\infty)}(\tilde{B}, p)$. To prove the theorem, we should show the existence of a linear bijection between the spaces $s_\alpha^{(\infty)}(\tilde{B}, p)$ and $\ell_\infty(p)$ for $0 < p_k \leq H < \infty$. With the notation of (2.2), define the transformations T from $s_\alpha^{(\infty)}(\tilde{B}, p)$ to $\ell_\infty(p)$ by $x \mapsto y = Tx$. The linearity of T is trivial. Further, it is obvious that $x = \theta$ whenever $Tx = \theta$ and hence T is injective.

Let $y = (y_k) \in \ell_\infty(p)$ and define the sequence $x = (x_k)$ by

$$(2.7) \quad x_k = \sum_{j=0}^k \frac{(-1)^{k-j} \alpha_j}{r_k} \prod_{i=j}^{k-1} \frac{s_i}{r_i}; \quad (k \in \mathbb{N}).$$

Then, we get that

$$\begin{aligned} g(x) &= \sup_{k \in \mathbb{N}} \left| \frac{r_k x_k + s_{k-1} x_{k-1}}{\alpha_k} \right|^{p_k/M} \\ &= \sup_{k \in \mathbb{N}} \left| \frac{r_k \sum_{j=0}^k \frac{(-1)^{k-j} \alpha_j}{r_k} \prod_{i=j}^{k-1} \frac{s_i}{r_i} + s_{k-1} \sum_{j=0}^{k-1} \frac{(-1)^{k-1-j} \alpha_j}{r_{k-1}} \prod_{i=j}^{k-2} \frac{s_i}{r_i}}{\alpha_k} \right|^{p_k/M} \\ &= \sup_{k \in \mathbb{N}} |y_k|^{p_k/M} = h_1(y) < \infty. \end{aligned}$$

Thus, we deduce that $x \in s_\alpha^{(\infty)}(\tilde{B}, p)$ and consequently T is surjective and is paranorm preserving. Hence, T is a linear bijection and this says us that the spaces $s_\alpha^{(\infty)}(\tilde{B}, p)$ and $\ell_\infty(p)$ are linearly isomorphic, as desired. \square

We shall quote some lemmas which are needed in proving related to the duals our theorems.

Lemma 2.3. [6, Theorem 5.1.1 with $q_n = 1$] $A \in (c_0(p) : \ell(q))$ if and only if

$$(2.8) \quad \sup_{K \in \mathcal{F}} \sum_n \left| \sum_{k \in K} a_{nk} B^{-1/p_k} \right| < \infty, \quad (\exists B \in \mathbb{N}_2).$$

Lemma 2.4. [6, Theorem 5.1.9 with $q_n = 1$] $A \in (c_0(p) : c(q))$ if and only if

$$(2.9) \quad \sup_{n \in \mathbb{N}} \sum_k |a_{nk}| B^{-1/p_k} < \infty \quad (\exists B \in \mathbb{N}_2),$$

$$(2.10) \quad \exists(\beta_k) \subset \mathbb{R} \ni \lim_{n \rightarrow \infty} |a_{nk} - \beta_k| = 0 \quad \text{for all } k \in \mathbb{N},$$

$$(2.11) \quad \exists(\beta_k) \subset \mathbb{R} \ni \sup_{n \in \mathbb{N}} \sum_k |a_{nk} - \beta_k| B^{-1/p_k} < \infty. \quad (\exists B \in \mathbb{N}_2)$$

Lemma 2.5. [6, Theorem 5.1.13 with $q_n = 1$] $A \in (c_0(p) : \ell_\infty(q))$ if and only if

$$(2.12) \quad \sup_{n \in \mathbb{N}} \sum_k |a_{nk}| B^{-1/p_k} < \infty. \quad (\exists B \in \mathbb{N}_2)$$

Lemma 2.6. [6, Theorem 5.1.0 with $q_n = 1$](i) Let $1 < p_k \leq H < \infty$ for all $k \in \mathbb{N}$. Then, $A \in (\ell(p) : \ell_1)$ if and only if there exists an integer $B > 1$ such that

$$(2.13) \quad \sup_{K \in \mathcal{F}} \sum_k \left| \sum_{n \in K} a_{nk} B^{-1} \right|^{p'_k} < \infty.$$

(ii) Let $0 < p_k \leq 1$ for all $k \in \mathbb{N}$. Then, $A \in (\ell(p) : \ell_1)$ if and only if

$$(2.14) \quad \sup_{K \in \mathcal{F}} \sup_{k \in \mathbb{N}} \left| \sum_{n \in K} a_{nk} \right|^{p_k} < \infty.$$

Lemma 2.7. [6, Theorem 1 (i)-(ii)] (i) Let $1 < p_k \leq H < \infty$ for all $k \in \mathbb{N}$. Then, $A \in (\ell(p) : \ell_\infty)$ if and only if there exists an integer $B > 1$ such that

$$(2.15) \quad \sup_{n \in \mathbb{N}} \sum_k |a_{nk} B^{-1}|^{p'_k} < \infty.$$

(ii) Let $0 < p_k \leq 1$ for all $k \in \mathbb{N}$. Then, $A \in (\ell(p) : \ell_\infty)$ if and only if

$$(2.16) \quad \sup_{n, k \in \mathbb{N}} |a_{nk}|^{p_k} < \infty.$$

Lemma 2.8. [6, Corollary for Theorem 1] Let $0 < p_k \leq H < \infty$ for all $k \in \mathbb{N}$. Then, $A \in (\ell(p) : c)$ if and only if (2.15), (2.16) hold, and

$$(2.17) \quad \lim_{n \rightarrow \infty} a_{nk} = \beta_k, \quad (k \in \mathbb{N})$$

also holds.

Theorem 2.9. Let $K^* = \{k \in \mathbb{N} : 0 \leq k \leq n\} \cap K$ for $K \in \mathcal{F}$ and $B \in \mathbb{N}_2$. Define the sets $S_1^\alpha(r, s), S_2^\alpha(r, s), S_3^\alpha(r, s), S_4^\alpha(r, s), S_5^\alpha(r, s), S_6^\alpha(r, s)$ and $S_7^\alpha(r, s)$ as follows:

$$\begin{aligned} S_1^\alpha(r, s) &= \bigcup_{B > 1} \left\{ a = (a_k) \in \omega : \sup_{K \in \mathcal{F}} \sum_n \left| \sum_{k \in K^*} \frac{(-1)^{n-k} \alpha_k}{r_n} \prod_{j=k}^{n-1} \frac{s_j}{r_j} a_n B^{-1/p_k} \right| < \infty \right\} \\ S_2^\alpha(r, s) &= \left\{ a = (a_k) \in \omega : \sum_n \left| \sum_{k=0}^n \frac{(-1)^{n-k} \alpha_k}{r_n} \prod_{j=k}^{n-1} \frac{s_j}{r_j} a_n \right| < \infty \right\} \\ S_3^\alpha(r, s) &= \bigcup_{B > 1} \left\{ a = (a_k) \in \omega : \sup_{n \in \mathbb{N}} \sum_{k=0}^n \left| \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j \right| B^{-1/p_k} < \infty \right\} \\ S_4^\alpha(r, s) &= \left\{ a = (a_k) \in \omega : \left| \sum_{j=k}^\infty \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j \right| < \infty \text{ for all } k \in \mathbb{N} \right\} \\ S_5^\alpha(r, s) &= \bigcup_{B > 1} \left\{ a = (a_k) \in \omega : \exists (\beta_k) \subset \mathbb{R} \ni \sup_{n \in \mathbb{N}} \sum_{k=0}^n \left| \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j - \beta_k \right| B^{-1/p_k} < \infty \right\} \\ S_6^\alpha(r, s) &= \left\{ a = (a_k) \in \omega : \exists \beta \in \mathbb{R} \ni \lim_{n \rightarrow \infty} \left| \sum_{k=0}^n \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j - \beta \right| = 0 \right\} \\ S_7^\alpha(r, s) &= \left\{ a = (a_k) \in \omega : \sup_{n \in \mathbb{N}} \left| \sum_{k=0}^n \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j \right| < \infty \right\} \end{aligned}$$

Then,

$$(i) \{s_\alpha^0(\tilde{B}, p)\}^\alpha = S_1^\alpha(r, s) \quad (ii) \{s_\alpha^{(c)}(\tilde{B}, p)\}^\alpha = S_1^\alpha(r, s) \cap S_2^\alpha(r, s)$$

$$(iii) \{s_\alpha^0(\tilde{B}, p)\}^\beta = S_3^\alpha(r, s) \cap S_4^\alpha(r, s) \cap S_5^\alpha(r, s)$$

$$(iv) \{s_\alpha^{(c)}(\tilde{B}, p)\}^\beta = \{s_\alpha^0(\tilde{B}, p)\}^\beta \cap S_6^\alpha(r, s)$$

$$(v) \{s_\alpha^0(\tilde{B}, p)\}^\gamma = S_3^\alpha(r, s) \quad (vi) \{s_\alpha^{(c)}(\tilde{B}, p)\}^\gamma = S_3^\alpha(r, s) \cap S_7^\alpha(r, s)$$

Proof. We give the proof for the space $s_\alpha^0(\tilde{B}, p)$. Let us take any $a = (a_n) \in \omega$ and define the matrix $C^\alpha = \{c_{nk}^\alpha(r, s)\}$ via the sequence $a = (a_n)$ by

$$c_{nk}^\alpha(r, s) = \begin{cases} \frac{(-1)^{n-k} \alpha_k}{r_n} \prod_{j=k}^{n-1} \frac{s_j}{r_j} a_n, & 0 \leq k \leq n, \\ 0, & k > n, \end{cases}$$

where $n, k \in \mathbb{N}$. Bearing in mind (2.7) we immediately derive that

$$(2.18) \quad a_n x_n = \sum_{k=0}^n \frac{(-1)^{n-k} \alpha_k}{r_n} \prod_{j=k}^{n-1} \frac{s_j}{r_j} a_n y_k = (C^\alpha y)_n; \quad (n \in \mathbb{N}).$$

We therefore observe by (2.18) that $ax = (a_n x_n) \in \ell_1$ whenever $x \in s_\alpha^0(\tilde{B}, p)$ if and only if $Cy \in \ell_1$ whenever $y \in c_0(p)$. This means that $a = (a_n) \in \{s_\alpha^0(\tilde{B}, p)\}^\alpha$ whenever $x = (x_n) \in s_\alpha^0(\tilde{B}, p)$ if and only if $C^\alpha \in (c_0(p) : \ell_1)$. Then, we derive by Lemma 2.3 that

$$\{s_\alpha^0(\tilde{B}, p)\}^\alpha = S_1^\alpha(r, s).$$

Consider the equation for $n \in \mathbb{N}$,

$$(2.19) \quad \begin{aligned} \sum_{k=0}^n a_k x_k &= \sum_{k=0}^n a_k \left[\sum_{j=0}^k \frac{(-1)^{k-j} \alpha_j}{r_k} \prod_{i=j}^{k-1} \frac{s_i}{r_i} y_j \right] \\ &= \sum_{k=0}^n \left[\sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j \right] y_k \\ &= (D^\alpha y)_n \end{aligned}$$

where $D^\alpha = \{d_{nk}^\alpha(r, s)\}$ is defined by

$$d_{nk}^\alpha = \begin{cases} \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j, & 0 \leq k \leq n, \\ 0, & k > n, \end{cases}$$

where $n, k \in \mathbb{N}$. Thus, we deduce from Lemma 2.4 with (2.19) that $ax = (a_k x_k) \in cs$ whenever $x = (x_k) \in s_\alpha^0(\tilde{B}, p)$ if and only if $D^\alpha y \in c$ whenever $y = (y_k) \in c_0(p)$. This means that $a = (a_n) \in \{s_\alpha^0(\tilde{B}, p)\}^\beta$ whenever $x = (x_n) \in s_\alpha^0(\tilde{B}, p)$ if and only if $D^\alpha \in (c_0(p) : c)$. Therefore we derive from Lemma 2.4 that

$$\{s_\alpha^0(\tilde{B}, p)\}^\beta = S_3^\alpha(r, s) \cap S_4^\alpha(r, s) \cap S_5^\alpha(r, s).$$

As this, we deduce from Lemma 2.5 with (2.19) that $ax = (a_k x_k) \in bs$ whenever $x = (x_k) \in s_\alpha^0(\tilde{B}, p)$ if and only if $D^\alpha y \in \ell_\infty$ whenever $y = (y_k) \in c_0(p)$. This means that $a = (a_n) \in \{s_\alpha^0(\tilde{B}, p)\}^\gamma$ whenever $x = (x_n) \in s_\alpha^0(\tilde{B}, p)$ if and only if $D^\alpha \in (c_0(p) : \ell_\infty)$. Therefore we obtain Lemma 2.5 that

$$\{s_\alpha^0(\tilde{B}, p)\}^\gamma = S_3^\alpha(r, s)$$

and this completes the proof. \square

Theorem 2.10. Let $K^* = \{k \in \mathbb{N} : 0 \leq k \leq n\} \cap K$ for $K \in \mathcal{F}$ and $B \in \mathbb{N}_2$. Define the sets $S_8^\alpha(r, s), S_9^\alpha(r, s), S_{10}^\alpha(r, s)$ and $S_{11}^\alpha(r, s)$ as follows:

$$\begin{aligned} S_8^\alpha(r, s) &= \bigcap_{B>1} \left\{ a = (a_k) \in \omega : \sup_{K \in \mathcal{F}} \sum_n \left| \sum_{k \in K^*} \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j B^{1/p_k} \right| < \infty \right\} \\ S_9^\alpha(r, s) &= \bigcap_{B>1} \left\{ a = (a_k) \in \omega : \sup_{n \in \mathbb{N}} \sum_{k=0}^n \left| \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j \right| B^{1/p_k} < \infty \right\} \\ S_{10}^\alpha(r, s) &= \bigcap_{B>1} \left\{ a = (a_k) \in \omega : \exists (\beta_k) \subset \mathbb{R} \ni \lim_{n \rightarrow \infty} \sum_{k=0}^n \left| \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j - \beta_k \right| B^{1/p_k} = 0 \right\} \\ S_{11}^\alpha(r, s) &= \bigcap_{B>1} \left\{ a = (a_k) \in \omega : \sup_{n \in \mathbb{N}} \sum_{k=0}^n \left| \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j \right| B^{1/p_k} < \infty \right\} \end{aligned}$$

Then,

$$\begin{aligned} (i) \quad \{s_\alpha^{(\infty)}(\tilde{B}, p)\}^\alpha &= S_8^\alpha(r, s) \\ (ii) \quad \{s_\alpha^{(\infty)}(\tilde{B}, p)\}^\beta &= S_9^\alpha(r, s) \cap S_{10}^\alpha(r, s) \\ (iii) \quad \{s_\alpha^{(\infty)}(\tilde{B}, p)\}^\gamma &= S_{11}^\alpha(r, s). \end{aligned}$$

Proof. This may be obtained in the similar way, as mentioned in the proof of Theorem 2.9 with Lemmas 2.6(i), 2.7(i), 2.8 instead of Lemmas 2.3-2.5. So, we omit the details. \square

Theorem 2.11. Let $K^* = \{k \in \mathbb{N} : 0 \leq k \leq n\} \cap K$ for $K \in \mathcal{F}$ and $B \in \mathbb{N}_2$. Define the sets $S_{12}^\alpha(r, s), S_{13}^\alpha(r, s), S_{14}^\alpha(r, s), S_{15}^\alpha(r, s)$ and $S_{16}^\alpha(r, s)$ as follows:

$$\begin{aligned} S_{12}^\alpha(r, s) &= \left\{ a = (a_k) \in \omega : \sup_{K \in \mathcal{F}} \sup_{k \in \mathbb{N}} \left| \sum_{n \in K^*} \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j \right|^{p_k} < \infty \right\} \\ S_{13}^\alpha(r, s) &= \bigcup_{B>1} \left\{ a = (a_k) \in \omega : \sup_{K \in \mathcal{F}} \sum_k \left| \sum_{n \in K} \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j B^{-1} \right|^{p'_k} < \infty \right\} \\ S_{14}^\alpha(r, s) &= \bigcup_{B>1} \left\{ a = (a_k) \in \omega : \sup_{n \in \mathbb{N}} \sum_{k=0}^n \left| \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j B^{-1} \right|^{p'_k} < \infty \right\} \\ S_{15}^\alpha(r, s) &= \left\{ a = (a_k) \in \omega : \sup_{n, k \in \mathbb{N}} \left| \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j \right|^{p_k} < \infty \right\} \\ S_{16}^\alpha(r, s) &= \left\{ a = (a_k) \in \omega : \lim_{n \rightarrow \infty} \sum_{j=k}^n \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_j \text{ exists} \right\} \end{aligned}$$

Then,

$$\begin{aligned} (i) \quad \{\ell_\alpha(\tilde{B}, p)\}^\alpha &= \begin{cases} S_{12}^\alpha(r, s), & 0 < p_k \leq 1 \\ S_{13}^\alpha(r, s), & 1 < p_k \leq H < \infty \end{cases} \\ (ii) \quad \{\ell_\alpha(\tilde{B}, p)\}^\gamma &= \begin{cases} S_{15}^\alpha(r, s), & 0 < p_k \leq 1 \\ S_{14}^\alpha(r, s), & 1 < p_k \leq H < \infty. \end{cases} \\ (iii) \quad \text{Let } 0 < p_k \leq H < \infty. \text{ Then,} \\ \{\ell_\alpha(\tilde{B}, p)\}^\beta &= S_{14}^\alpha(r, s) \cap S_{15}^\alpha(r, s) \cap S_{16}^\alpha(r, s). \end{aligned}$$

Proof. This may be obtained in the similar way, as mentioned in the proof of Theorem 2.9 with Lemmas 2.6(ii), 2.7(ii), 2.8 instead of Lemmas 2.3-2.5. So, we omit the details. \square

Now, we may give the sequence of the points of the spaces $s_\alpha^0(\tilde{B}, p)$, $\ell_\alpha(\tilde{B}, p)$ and $s_\alpha^{(c)}(\tilde{B}, p)$ which forms a Schauder basis for those spaces. Because of the isomorphism T , defined in the proof of Theorem 2.2, between the sequence spaces $s_\alpha^0(\tilde{B}, p)$ and $c_0(p)$, $\ell_\alpha(\tilde{B}, p)$ and $\ell(p)$, $s_\alpha^{(c)}(\tilde{B}, p)$ and $c(p)$ is onto, the inverse image of the basis of the spaces $c_0(p)$, $\ell(p)$ and $c(p)$ is the basis for our new spaces $s_\alpha^0(\tilde{B}, p)$, $\ell_\alpha(\tilde{B}, p)$ and $s_\alpha^{(c)}(\tilde{B}, p)$, respectively. Therefore, we have:

Theorem 2.12. *Let $\mu_k = (\tilde{T}x)_k$ for all $k \in \mathbb{N}$. We define the sequence $b^{(k)} = \{b_n^{(k)}\}_{n \in \mathbb{N}}$ for every fixed $k \in \mathbb{N}$ by*

$$b_n^{(k)} = \begin{cases} \frac{(-1)^{n-k} \alpha_k}{r_n} \prod_{j=k}^{n-1} \frac{s_j}{r_j}, & n \geq k, \\ 0, & n < k. \end{cases}$$

Then,

(a) *The sequence $\{b^{(k)}\}_{k \in \mathbb{N}}$ is a basis for the space $s_\alpha^0(\tilde{B}, p)$ and any $x \in s_\alpha^0(\tilde{B}, p)$ has a unique representation in the form*

$$x = \sum_k \mu_k b^{(k)}.$$

(b) *The sequence $\{b^{(k)}\}_{k \in \mathbb{N}}$ is a basis for the space $\ell_\alpha(\tilde{B}, p)$ and any $x \in \ell_\alpha(\tilde{B}, p)$ has a unique representation in the form*

$$x = \sum_k \mu_k b^{(k)}.$$

(c) *The set $\{z, b^{(k)}\}$ is a basis for the space $s_\alpha^{(c)}(\tilde{B}, p)$ and any $x \in s_\alpha^{(c)}(\tilde{B}, p)$ has a unique representation in the form*

$$x = lz + \sum_k (\mu_k - l) b^{(k)}$$

where $l = \lim_{k \rightarrow \infty} (\tilde{T}x)_k$ and $z = (z_k)$ with

$$z_k = \sum_{j=0}^k \frac{(-1)^{k-j} \alpha_j}{r_k} \prod_{i=j}^{k-1} \frac{s_i}{r_i}.$$

3. SOME MATRIX MAPPINGS ON THE SEQUENCE SPACES $s_\alpha^0(\tilde{B}, p)$, $s_\alpha^{(c)}(\tilde{B}, p)$, $s_\alpha^{(\infty)}(\tilde{B}, p)$ AND $\ell_\alpha(\tilde{B}, p)$

In this section, we characterize some matrix mappings on the spaces $s_\alpha^0(\tilde{B}, p)$, $s_\alpha^{(c)}(\tilde{B}, p)$, $s_\alpha^{(\infty)}(\tilde{B}, p)$ and $\ell_\alpha(\tilde{B}, p)$. Firstly, we may give the following theorem which is useful for deriving the characterization of the certain matrix classes.

Theorem 3.1. [36, Theorem 4.1] *Let λ be an FK-space, U be a triangle, V be its inverse and μ be arbitrary subset of ω . Then we have $A \in (\lambda_U : \mu)$ if and only if*

$$(3.1) \quad E^{(n)} = (e_{mk}^{(n)}) \in (\lambda : c) \quad \text{for all } n \in \mathbb{N}$$

and

$$(3.2) \quad E = (e_{nk}) \in (\lambda : \mu)$$

where

$$e_{mk}^{(n)} = \begin{cases} \sum_{j=k}^m a_{nj} v_{jk}, & 0 \leq k \leq m, \\ 0, & k > m, \end{cases}$$

and

$$e_{nk} = \sum_{j=k}^{\infty} a_{nj} v_{jk} \quad \text{for all } k, m, n \in \mathbb{N}.$$

Now, we may quote our theorems on the characterization of some matrix classes concerning with the sequence spaces $s_\alpha^0(\tilde{B}, p)$, $s_\alpha^{(c)}(\tilde{B}, p)$ and $s_\alpha^{(\infty)}(\tilde{B}, p)$. The necessary and sufficient conditions characterizing the matrix mappings between the sequence spaces of Maddox are determined by Grosse-Erdmann [6]. Let N and K denote the finite subset of \mathbb{N} , L and M also denote the natural numbers. Prior to giving the theorems, let us suppose that (q_n) is a non-decreasing bounded sequence of positive numbers and consider the following conditions:

$$(3.3) \quad \lim_{m \rightarrow \infty} \sum_{j=k}^m \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_{nj} = e_{nk},$$

$$(3.4) \quad \forall L, \quad \sum_k |e_{nk}| L^{1/p_k} < \infty,$$

$$(3.5) \quad \exists (\beta_k) \subset \mathbb{R} \ni \lim_{m \rightarrow \infty} \left| \sum_{j=k}^m \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_{nj} - \beta_k \right| = 0 \quad \text{for all } k \in \mathbb{N},$$

$$(3.6) \quad \exists M, \quad \sup_{m \in \mathbb{N}} \sum_{k=0}^m \left| \sum_{j=k}^m \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_{nj} \right| M^{-1/p_k} < \infty,$$

$$(3.7) \quad \forall L, \exists M, \sup_{m \in \mathbb{N}} \sum_{k=0}^m \left| \sum_{j=k}^m \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_{nj} \right| L^{1/q_n} M^{-1/p_k} < \infty,$$

$$(3.8) \quad \lim_{m \rightarrow \infty} \sum_k \left| \sum_{j=k}^m \frac{(-1)^{j-k} \alpha_k}{r_j} \prod_{i=k}^{j-1} \frac{s_i}{r_i} a_{nj} - \beta \right| = 0,$$

$$(3.9) \quad \forall L, \quad \sup_{n \in \mathbb{N}} \sum_k |e_{nk}| L^{1/p_k} < \infty,$$

$$(3.10) \quad \lim_{n \rightarrow \infty} e_{nk} = \beta_k \quad \text{for all } k \in \mathbb{N},$$

$$(3.11) \quad \forall L, \quad \lim_{n \rightarrow \infty} \sum_k |e_{nk}| L^{1/p_k} < \infty,$$

$$(3.12) \quad \forall L, \quad \lim_{n \rightarrow \infty} \sum_k |e_{nk}| L^{1/p_k} = 0,$$

$$(3.13) \quad \exists M, \quad \sup_{n \in \mathbb{N}} \left(\sum_{k \in K} |e_{nk}| M^{-1/p_k} \right)^{q_n} < \infty,$$

$$(3.14) \quad \lim_{n \rightarrow \infty} |e_{nk}|^{q_n} = 0, \quad \text{for all } k \in \mathbb{N},$$

$$(3.15) \quad \forall L, \exists M, \quad \sup_{n \in \mathbb{N}} \sum_k |e_{nk}| L^{1/q_n} M^{-1/p_k} < \infty,$$

$$(3.16) \quad \lim_{n \rightarrow \infty} |e_{nk} - \beta_k|^{q_n} = 0, \quad \text{for all } k \in \mathbb{N},$$

$$(3.17) \quad \exists M, \quad \sup_{n \in \mathbb{N}} \sum_k |e_{nk}| M^{-1/p_k} < \infty,$$

$$(3.18) \quad \forall L, \exists M, \sup_{n \in \mathbb{N}} \sum_k |e_{nk} - \beta_k| L^{1/q_n} M^{-1/p_k} < \infty,$$

$$(3.19) \quad \sup_{n \in \mathbb{N}} \left| \sum_k e_{nk} \right|^{q_n} < \infty,$$

$$(3.20) \quad \lim_{n \rightarrow \infty} \left| \sum_k e_{nk} \right|^{q_n} = 0,$$

$$(3.21) \quad \lim_{n \rightarrow \infty} \left| \sum_k e_{nk} - \beta \right|^{q_n} = 0,$$

Theorem 3.2. (i) $A \in (s_\alpha^{(\infty)}(\tilde{B}, p) : \ell_\infty)$ if and only if (3.3), (3.4) and (3.9) hold.

(ii) $A \in (s_\alpha^{(\infty)}(\tilde{B}, p) : c)$ if and only if (3.3), (3.4), (3.10) and (3.11) hold.

(iii) $A \in (s_\alpha^{(\infty)}(\tilde{B}, p) : c_0)$ if and only if (3.3), (3.4) and (3.12) hold.

Theorem 3.3. (i) $A \in (s_\alpha^0(\tilde{B}, p) : \ell_\infty(q))$ if and only if (3.5), (3.6), (3.7) and (3.13) hold.

(ii) $A \in (s_\alpha^0(\tilde{B}, p) : c_0(q))$ if and only if (3.5), (3.6), (3.7), (3.14) and (3.15) hold.

(iii) $A \in (s_\alpha^0(\tilde{B}, p) : c(q))$ if and only if (3.5), (3.6), (3.7), (3.16), (3.17) and (3.18) hold.

Theorem 3.4. (i) $A \in (s_\alpha^{(c)}(\tilde{B}, p) : \ell_\infty(q))$ if and only if (3.5), (3.6), (3.7), (3.8), (3.13) and (3.19) hold.

(ii) $A \in (s_\alpha^{(c)}(\tilde{B}, p) : c_0(q))$ if and only if (3.5), (3.6), (3.7), (3.8), (3.14), (3.15) and (3.20) hold.

(iii) $A \in (s_\alpha^{(c)}(\tilde{B}, p) : c(q))$ if and only if (3.5), (3.6), (3.7), (3.8), (3.16), (3.17), (3.18) and (3.21) hold.

4. α -CORE

Using the convergence domain of the matrix $\tilde{T} = \{t_{nk}^\alpha(r, s)\}$, the new sequence spaces $s_\alpha^0(\tilde{B})$ and $s_\alpha^{(c)}(\tilde{B})$ have been constructed and their some properties have been investigated in [44]. In this section we will consider the sequences with complex entries and by $\ell_\infty(\mathbb{C})$ denote the space of all bounded complex valued sequences.

Following Knopp, a core theorem is characterized a class of matrices for which the core of the transformed sequence is included by the core of the original sequence. For example Knopp Core Theorem [14, p. 138] states that $\mathcal{K} - \text{core}(Ax) \subseteq \mathcal{K} - \text{core}(x)$ for all real valued sequences x whenever A is a positive matrix in the class $(c : c)_{\text{reg}}$.

Here, we will define $\alpha - \text{core}$ of a complex valued sequence and characterize the class of matrices to yield $\alpha - \text{core}(Ax) \subseteq \mathcal{K} - \text{core}(x)$ and $\alpha - \text{core}(Ax) \subseteq st - \text{core}(x)$ for all $x \in \ell_\infty(\mathbb{C})$.

Now, let us write

$$\tau_n(x) = \frac{r_n x_n + s_{n-1} x_{n-1}}{\alpha_n}$$

where $n \in \mathbb{N}$. Then, we can define $\alpha - \text{core}$ of a complex sequence as follows:

Definition 4.1. Let H_n be the least closed convex hull containing $\tau_n(x), \tau_{n+1}(x), \tau_{n+2}(x), \dots$. Then, $\alpha - \text{core}$ of x is the intersection of all H_n , i.e.,

$$\alpha - \text{core}(x) = \bigcap_{n=1}^{\infty} H_n.$$

Note that, actually, we define $\alpha - \text{core}$ of x by the $\mathcal{K} - \text{core}$ of the sequence $(\tau_n(x))$. Hence, we can construct the following theorem which is an analogue of $\mathcal{K} - \text{core}$, [15]:

Theorem 4.2. For any $z \in \mathbb{C}$, let

$$G_x(z) = \left\{ \omega \in \mathbb{C} : |\omega - z| \leq \limsup_n |\tau_n(x) - z| \right\}.$$

Then, for any $x \in \ell_\infty$,

$$\alpha - \text{core}(x) = \bigcap_{z \in \mathbb{C}} G_x(z).$$

Now, we prove some lemmas which will be useful to the main results of this section. To do these, we need to characterize the classes $(c : s_\alpha^{(c)}(\tilde{B}))_{\text{reg}}$ and $(st(A) \cap \ell_\infty : s_\alpha^{(c)}(\tilde{B}))_{\text{reg}}$. For brevity, in what follows we write \tilde{b}_{nk} in place of

$$\frac{r_n b_{nk} + s_{n-1} b_{n-1,k}}{\alpha_n}; \quad (n \geq 1).$$

Lemma 4.3. $B \in (\ell_\infty : s_\alpha^{(c)}(\tilde{B}))$ if and only if

$$(4.1) \quad \|B\| = \sup_n \sum_k |\tilde{b}_{nk}| < \infty,$$

$$(4.2) \quad \lim_n \tilde{b}_{nk} = \beta_k \text{ for each } k,$$

$$(4.3) \quad \lim_n \sum_k |\tilde{b}_{nk} - \beta_k| = 0.$$

Lemma 4.4. $B \in (c : s_\alpha^{(c)}(\tilde{B}))_{\text{reg}}$ if and only if (4.1) and (4.2) of the Lemma 4.3 hold with $\beta_k = 0$ for all $k \in \mathbb{N}$ and

$$(4.4) \quad \lim_n \sum_k \tilde{b}_{nk} = 1.$$

Lemma 4.5. $B \in (st(A) \cap \ell_\infty : s_\alpha^{(c)}(\tilde{B}))_{\text{reg}}$ if and only if $B \in (c : s_\alpha^{(c)}(\tilde{B}))_{\text{reg}}$ and

$$(4.5) \quad \lim_n \sum_{k \in E} |\tilde{b}_{nk}| = 0$$

for every $E \subset \mathbb{N}$ with $\delta_A(E) = 0$.

Proof. Because of $c \subset st \cap \ell_\infty$, $B \in (c : s_\alpha^{(c)}(\tilde{B}))_{\text{reg}}$. Now, for any $x \in \ell_\infty$ and a set $E \subset \mathbb{N}$ with $\delta(E) = 0$, let us define the sequence $z = (z_k)$ by

$$z_k = \begin{cases} x_k, & k \in E \\ 0, & k \notin E. \end{cases}$$

Then, since $z \in st_0$, $Az \in s_\alpha^{(0)}(\tilde{B})$, where $s_\alpha^{(0)}(\tilde{B})$ is the space of sequences which the \tilde{T} -transforms of them in c_0 . Also, since

$$\sum_k \tilde{b}_{nk} z_k = \sum_{k \in E} \tilde{b}_{nk} x_k,$$

the matrix $D = (d_{nk})$ defined by $d_{nk} = \tilde{b}_{nk}$ ($k \in E$) and $d_{nk} = 0$ ($k \notin E$) is in the class $(\ell_\infty : s_\alpha^{(c)}(\tilde{B}))$. Hence, the necessity of (4.5) follows from Lemma 4.3.

Conversely, let $x \in st(A) \cap \ell_\infty$ with $st_A - \lim x = l$. Then, the set E defined by $E = \{k : |x_k - l| \geq \varepsilon\}$ has density zero and $|x_k - l| \leq \varepsilon$ if $k \notin E$. Now, we can write

$$(4.6) \quad \sum_k \tilde{b}_{nk} x_k = \sum_k \tilde{b}_{nk} (x_k - l) + l \sum_k \tilde{b}_{nk}.$$

Since

$$\left| \sum_k \tilde{b}_{nk} (x_k - l) \right| \leq \|x\| \sum_{k \in E} |\tilde{b}_{nk}| + \varepsilon \cdot \|B\|,$$

letting $n \rightarrow \infty$ in (4.6) and using (4.4) with (4.5), we have

$$\lim_n \sum_k \tilde{b}_{nk} x_k = l.$$

This implies that $B \in (st(A) \cap \ell_\infty : s_\alpha^{(c)}(\tilde{B}))_{reg}$ and the proof is completed. \square

Now, we may give some inclusion theorems. Firstly, we need a lemma.

Lemma 4.6. [16, Corollary 12] *Let $A = (a_{nk})$ be a matrix satisfying $\sum_k |a_{nk}| < \infty$ and $\lim_n a_{nk} = 0$. Then, there exists an $y \in \ell_\infty$ with $\|y\| \leq 1$ such that*

$$\limsup_n \sum_k a_{nk} y_k = \limsup_n \sum_k |a_{nk}|.$$

Theorem 4.7. *Let $B \in (c : s_\alpha^{(c)}(\tilde{B}))_{reg}$. Then, $\alpha - core(Bx) \subseteq \mathcal{K} - core(x)$ for all $x \in \ell_\infty$ if and only if*

$$(4.7) \quad \lim_n \sum_k |\tilde{b}_{nk}| = 1.$$

Proof. Since $B \in (c : s_\alpha^{(c)}(\tilde{B}))_{reg}$, the matrix $\hat{B} = (\tilde{b}_{nk})$ is satisfy the conditions of Lemma 4.6. So, there exists a $y \in \ell_\infty$ with $\|y\| \leq 1$ such that

$$\left\{ \omega \in \mathbb{C} : |\omega| \leq \limsup_n \sum_k \tilde{b}_{nk} y_k \right\} = \left\{ \omega \in \mathbb{C} : |\omega| \leq \limsup_n \sum_k |\tilde{b}_{nk}| \right\}.$$

On the other hand, since $\mathcal{K} - core(y) \subseteq B_1(0)$, by the hypothesis

$$\left\{ \omega \in \mathbb{C} : |\omega| \leq \limsup_n \sum_k |\tilde{b}_{nk}| \right\} \subseteq B_1(0) = \{\omega \in \mathbb{C} : |\omega| \leq 1\}$$

which implies (4.7).

Conversely, let $\omega \in \alpha - core(Bx)$. Then, for any given $z \in \mathbb{C}$, we can write

$$(4.8) \quad \begin{aligned} |\omega - z| &\leq \limsup_n |\tau_n(Bx) - z| \\ &= \limsup_n \left| z - \sum_k \tilde{b}_{nk} x_k \right| \\ &\leq \limsup_n \left| \sum_k \tilde{b}_{nk} (z - x_k) \right| + \limsup_n |z| \left| 1 - \sum_k \tilde{b}_{nk} \right| \\ &= \limsup_n \left| \sum_k \tilde{b}_{nk} (z - x_k) \right|. \end{aligned}$$

Now, let $\limsup_k |x_k - z| = l$. Then, for any $\varepsilon > 0$, $|x_k - z| \leq l + \varepsilon$ whenever $k \geq k_0$. Hence, one can write that

$$(4.9) \quad \begin{aligned} \left| \sum_k \tilde{b}_{nk} (z - x_k) \right| &= \left| \sum_{k < k_0} \tilde{b}_{nk} (z - x_k) + \sum_{k \geq k_0} \tilde{b}_{nk} (z - x_k) \right| \\ &\leq \sup_k |z - x_k| \sum_{k < k_0} |\tilde{b}_{nk}| + (l + \varepsilon) \sum_{k \geq k_0} |\tilde{b}_{nk}| \\ &\leq \sup_k |z - x_k| \sum_{k < k_0} |\tilde{b}_{nk}| + (l + \varepsilon) \sum_k |\tilde{b}_{nk}|. \end{aligned}$$

Therefore, applying \limsup_n under the light of the hypothesis and combining (4.8) with (4.9), we have

$$|\omega - z| \leq \limsup_n \left| \sum_k \tilde{b}_{nk}(z - x_k) \right| \leq l + \varepsilon$$

which means that $\omega \in \mathcal{K} - \text{core}(x)$. This completes the proof. \square

Theorem 4.8. *Let $B \in (st(A) \cap \ell_\infty : s_\alpha^{(c)}(\tilde{B}))_{reg}$. Then, $\alpha - \text{core}(Bx) \subseteq st_A - \text{core}(x)$ for all $x \in \ell_\infty$ if and only if (4.7) holds.*

Proof. Since $st_A - \text{core}(x) \subseteq \mathcal{K} - \text{core}(x)$ for any sequence x [19], the necessity of the condition (4.7) follows from Theorem 4.7.

Conversely, take $\omega \in \alpha - \text{core}(Bx)$. Then, we can write again (4.8). Now; if $st_A - \limsup |x_k - z| = s$, then for any $\varepsilon > 0$, the set E defined by $E = \{k : |x_k - z| > s + \varepsilon\}$ has density zero, (see [19]). Now, we can write

$$\begin{aligned} \left| \sum_k \tilde{b}_{nk}(z - x_k) \right| &= \left| \sum_{k \in E} \tilde{b}_{nk}(z - x_k) + \sum_{k \notin E} \tilde{b}_{nk}(z - x_k) \right| \\ &\leq \sup_k |z - x_k| \sum_{k \in E} |\tilde{b}_{nk}| + (s + \varepsilon) \sum_{k \notin E} |\tilde{b}_{nk}| \\ &\leq \sup_k |z - x_k| \sum_{k \in E} |\tilde{b}_{nk}| + (s + \varepsilon) \sum_k |\tilde{b}_{nk}|. \end{aligned}$$

Thus, applying the operator \limsup_n and using the condition (4.7) with (4.5), we get that

$$(4.10) \quad \limsup_n \left| \sum_k \tilde{b}_{nk}(z - x_k) \right| \leq s + \varepsilon.$$

Finally, combining (4.8) with (4.10), we have

$$|\omega - z| \leq st_A - \limsup_k |x_k - z|$$

which means that $\omega \in st_A - \text{core}(x)$ and the proof is completed. \square

As a consequence of Theorem 4.8, we have

Corollary 4.9. *Let $B \in (st \cap \ell_\infty : s_\alpha^{(c)}(\tilde{B}))_{reg}$. Then, $\alpha - \text{core}(Bx) \subseteq st - \text{core}(x)$ for all $x \in \ell_\infty$ if and only if (4.7) holds.*

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